Imperfect capital markets and human capital accumulation

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Abstract

In this paper we argue that income uncertainty depresses human capital investment. This problem can be avoided if financial markets are complete. Completeness is more likely to hold when the individuals face aggregate risks than when the main source of risk is idiosyncratic.

*Jel* classification: *G1; I2; J24*


1 Introduction

During the last fifty years, economists have devoted a great deal of effort to developing the concept of human capital. The concept of human capital was first introduced by Schultz (1961), who defines human capital investment as consisting of education and on-the-job training. Investment in human capital is costly at the time when it is undertaken, but it increases an individual’s earning capacity and therefore increases future consumption. Becker (1962) expands the definition of human capital to include apart from education and on-the-job training, such variables as investment in health, for example via medical care or proper diet, and acquiring information (Becker, 1962, 1965).

The early models of human capital were quite complicated and did not allow to perform definite comparative static exercises. The first tractable model was developed by Ben-Porath (1967) which latter Krebs (2003) and Singh (2010) augmented the basis model with new variables in their study of human capital model in incomplete-market. It followed Schultz by treating human capital as combination of education and on-the-job training and made some definite predictions about the effect of education and experience on earning. On the basis of these theoretical contribution Mincer (1997) developed the concept of earning equation, suitable for empirical estimation and applied
it to explain investment in education, income inequality, and life-time earning profiles. Some further papers, followed earlier Becker’s contributions to develop tractable human capital to include multiple dimensions, such as education and health. The most prominent papers that took this approach are Grossman (1970, 1972) and McDonald and Roberts (2002).

Although Krebs (2003) and Singh (2010) mentioned about the uncertainty and idiosyncratic of human capital investment, similar to all the other models (Becker, 1962; Ben-Porath, 1967; Grossman, 1970, 1972; McDonald & Roberts, 2002), however, assume that the returns to investment to human capital are certain. In practice, an individual faces a considerable uncertainty when making human capital investment decisions. Skills and knowledge learned in the college, for example; the ability to apply particular software can become obsolete in the future. With the accelerating speed of technological development, the individual maybe unable to adapt. Only in Jacobs (2007), the issue of uncertainty in human capital investment outcome has been address in the model. Because of the uncertainty in human capital investment, unemployment insurance were introduced to reduce income risk faced by risk-averse workers (Brown & Kaufold, 1988). In this paper, with different approach from Jacobs (2007), we included the uncertainty in our
two period life-cycle models and add in insurance as an option to reduce the idiosyncratic human capital risk, in which investment in human capital takes place in period one and earning are realised in period two. The expected earning increase in the investment in human capital, but actual earning are uncertain. We show that the investment in human capital in the case when a full income insurance is possible exceeds investment in the case where no income insurance exists.

Our next insight is that even if direct income insurance is not available, financial markets can be used to diversify risk and transfer wealth from more favourable states of nature to the less favourable ones. For a review on the functions and operation of financial markets, see Thangavelu and Ang Beng Jiunn (2004). We argue that if financial markets are complete, then the individual effectively has access to a full income insurance. An important question is, how likely are financial markets to be complete? By definition, the markets are complete if the number of independent financial assets\(^1\) equals the number of different states of nature. This definition requires two things, first there should be enough assets, second their payoffs should be sensitive enough to the states of the world. From the point of view of individual, the

\(^{1}\text{The set of assets can include both primary assets and derivatives}\)
situation when agent is employed and can make full use of its human capital, and the situation when agent is unemployed or employed at the job that does not make use of its skills are different states of the world. Whether the market will treat these states differently, depends whether different employment outcomes are due to aggregate risk or idiosyncratic risk. In the first case the market will treat these states differently, thus properly functioning financial markets can provide income insurance and boost the investment to human capital. In the second case, market will not be able to distinguish between individually relevant states and therefore even well functioning financial markets will likely remain incomplete from the individual's point of view. As a result, direct income insurance is needed to increase investment to human capital. Another approach to managing risk, income contingent loans, is discussed by Chapman (2006).

2 A two-period model with a possibility of direct insurance

Assume that an individual lives for at most two periods, in which the first period is when agent invest in education and the second period is when agent
start to receive human capital returns. The preferences of the individual over a consumption stream \((c_1, c_2)\) are given by

\[
U(c_1, c_2) = u(c_1) + \delta u(c_2),
\]

(1)

where \(u(\cdot)\) is twice differentiable and strictly concave,

\[
\lim_{c \to +0} u'(c) = \infty
\]

and \(\delta \in (0, 1)\). Assume that there are no credit market imperfection and the individual is born with assets \(A\), which can include both physical and human capital. In period one, the individual decides how to divide their assets between current consumption, \(c_1\), investment in the physical capital, \(s\), and investment in human capital, \(x\), and income insurance, \(d\). Therefore,

\[
A = c_1 + s + d + x.
\]

(2)
Investment in physical capital earns returns at rate $r$ with certainty. Investment in human capital produces level of skills, $h$, where

$$h = x^\alpha$$

for some $\alpha \in (0, 1)$. This human capital will produce earnings $w = h$ with probability $p$, and become obsolete and produce no earnings with the complimentary probability. If agents human capital produces no earnings, they are entitled to income protection payment, $z$. Since period two is assumed to be the terminal period of the individual’s life the agent will consume its entire second period income, $y_2$, which is a random variable

$$y_2 = \begin{cases} s(1 + r) + x^\alpha, & \text{with probability } p \\ s(1 + r) + z, & \text{with probability } 1 - p \end{cases}$$

The individual solves

$$\max[u(c_1) + \delta Eu(y_2)]$$

$$s.t. \quad A = c_1 + s + d + x$$

$$d = d(z)$$
where equation (6) is the first period budget constraint and equation (7) specifies the insurance company’s policy, i.e. it specifies the insurance premium as a function of the coverage. Let us consider two possible scenarios: first, a scenario when the insurance is unavailable and then the one when the insurance is possible.

2.1 Income protection is possible

In this Section we will assume that capital markets are complete, so the individual can buy income insurance. In this case the problem is:

\[
\max_{s,x,z} [u(A - s - x - d(z)) + \delta[pu(s(1 + r) + x^\alpha) + (1 - p)u(s(1 + r) + z)].
\]  

The first order conditions are:

\[
\begin{align*}
\begin{cases}
u'(A - s - x - d) = \alpha x^{\alpha-1} p \delta u'(s(1 + r) + x^\alpha) \\
u'(A - s - x - d) = \delta (1 + r)[pu'(s(1 + r) + x^\alpha) + (1 - p)u'(s(1 + r) + z)] \\
u'(A - s - x - d) d'(z) = \delta (1 - p)u'(s(1 + r) + z). 
\end{cases}
\end{align*}
\]  

The first order conditions allow us to arrive at the following general result:

**Proposition 1** Assume that \( u(\cdot) \) is differentiable and strictly concave, in-
surance is available at actuarially fair odds, i.e.

\[(1 + r)d = (1 - p)z, \quad (10)\]

and the discount rate equals the rate of time preferences

\[\delta(1 + r) = 1. \quad (11)\]

Then consumption in the second period does not depend on realization of uncertainty and

\[c_1 = c_2 = Ey_2. \quad (12)\]

The investment to human capital, \(x_F\), insurance premium, and insurance amount are given by:

\[\alpha px_F^{\alpha - 1} = 1 + r, \quad z = x_F^\alpha, \quad d = \frac{(1 - p)x_F^\alpha}{1 + r}. \quad (13)\]

\textbf{Proof} The last of conditions (9) and equations (10), (11) imply that

\[u'(A - s - x - d) = u'(c_1) = u'(s(1 + r) + z). \quad (14)\]
Now, the second equation of system (9) and (11) imply that

\[ u'(A - s - x - d) = u'(c_1) = u'(s(1 + r) + x^\alpha). \tag{15} \]

But, since \( u(\cdot) \) is differentiable and strictly concave, \( u'(\cdot) \) is strictly decreasing and hence

\[ c_1 = s(1 + r) + z = s(1 + r) + x^\alpha = c_2. \tag{16} \]

Therefore, under the conditions of the Proposition the individual achieves perfect consumption smoothing. Finally, taking into account (16) and the second equation in the system (9) one obtains:

\[ u'(A - s - x_F - d) = \frac{\alpha px_F^{\alpha-1}}{1 + r} u'(A - s - x_F - d), \tag{17} \]

which, together with (16) and (10), implies (13). ■

The intuition behind the condition (13) is straightforward. It claims that the expected return on human capital equals the return on physical capital. Since one can purchase a complete income insurance at actuarially fair odds, the riskiness of the human capital investment does not matter.
2.2 No income protection is possible

In this Section we will assume that capital markets are incomplete and as a result no income protection is possible. This will imply that \( d = z = 0 \) and the individual's problem becomes:

\[
\max_{s,x} [u(A - s - x) + \delta [pu(s(1 + r) + x^\alpha) + (1 - p)u(s(1 + r))]]. \tag{18}
\]

The first order conditions are:

\[
\begin{align*}
    u'(A - s - x) &= \alpha x^{\alpha-1} p \delta u'(s(1 + r) + x^\alpha) \\
    u'(A - s - x) &= \delta (1 + r) [pu'(s(1 + r) + x^\alpha) + (1 - p)u'(s(1 + r))] \\
\end{align*}
\tag{19}
\]

As in the previous Section, let \( x_F \) denote investment in human capital under full insurance and \( x_N \) denotes investment in human capital under no insurance.

**Proposition 2.** Under conditions of Proposition 1, \( x_N < x_F \).

**Proof.** The first order conditions (19) imply

\[
\alpha x_N^{\alpha-1} p \delta u'(s(1 + r) + x_N^\alpha) = [pu'(s(1 + r) + x_N^\alpha) + (1 - p)u'(s(1 + r))]. \tag{20}
\]
Here we have taken into account that $\delta(1 + r) = 1$. Let us assume, to the contrary of the statement of the Proposition, that $x_N \geq x_F$. Then, since $\alpha \in (0, 1)$ this implies:

$$\alpha px_N^{\alpha - 1} \leq \alpha px_F^{\alpha - 1} = 1 + r$$

(21)

and equation (20) implies:

$$u'(s(1 + r) + x_N) \geq pu'(s(1 + r) + x_N) + (1 - p)u'(s(1 + r)),$$

(22)

which can be re-written as:

$$u'(s(1 + r) + x_N) \geq u'(s(1 + r)).$$

(23)

Concavity of $u(\cdot)$ now implies:

$$s(1 + r) + x_N^\alpha \leq s(1 + r),$$

(24)

which is a contradiction, since we assumed $x_N \geq x_F > 0$.

Intuitively, investment in human capital is now riskier than investment
in the physical capital. Therefore, the investor requires higher return. Since
the rate of return to the investment in human capital is decreasing in the
amount investment, the optimal amount should be smaller than under full in-
surance to generate higher returns. According to Brown and Kaufold (1988),
unemployment insurance had been introduced to reduce income risk. Re-
duction in income risk reduces riskiness in human capital investment and
therefore increases the level of investment in human capital, which in turn
fuels economic growth. The last phenomenon was documented by Krebs
(2003). These observation are consistent with Proposition 2.

3 Financial markets and income protection

Assume there are \( S \) states of nature. State of nature includes all payoff
relevant information for agents in the economy. For example, if there are \( N \)
individuals and each of them can be either employed or unemployed, reali-
sation of employment outcome is independent across agents, and there is no
other source of uncertainty, there are \( 2^N \) states of nature, each correspond-
ing to a string of 0s and 1s of length \( N \), with 1 in \( i^{th} \) position if agent \( i \) is
employed and 0 otherwise. If the economy is similar to one described above,
but employment status is determined by an aggregate shock, which can take two values: $G$ when everybody is employed, and $L$ when odd-numbered individuals are unemployed and even numbered individuals are employed than there are just two states of nature. Let us assume that there are $K$ different assets and let $R$ be $S \times K$ matrix of their returns. Its generic element, $r_{sk}$, is the return to asset $k$ in states $s$.\textsuperscript{2}

**Definition 1.** An asset structure is called complete if $\text{rank}(R) = S$.

Note that the necessary condition for completeness of the asset structure is $K \geq S$. This condition is, however, not sufficient. For example, let $S = 2$ and

$$R = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}.$$\textsuperscript{(25)}

Note that $K = 2 = S$, but $\text{rank}(R) = 1$, since its columns are dependent. One way to interpret dependence is to say that from the point of view of the market to states look the same. In the first example of this Section, most of $2^N$ different states will look the same to the market. For example, let us assume that $K \leq N$ individuals have the same qualifications. Then any two states of nature in which $K_1$ of them are employed and $K - K_1$ are

\textsuperscript{2}In this Section we measure returns in dollars rather than in percentages
unemployed will look the same from the point of view of the market, i.e. corresponding rows in the matrix $R$ will be the same. This example shows that complete market structure is unlikely to develop when the individuals face idiosyncratic risk. In the second case, however, complete market structure is quite easy to obtain. Suppose be have just two assets the risk free asset and another asset $r_2 = (r_{12}, r_{22})$, with $r_{12} \neq r_{22}$. The last assumption is easy to justify, since it is reasonable to assume that the return to the risky asset depends on the level of unemployment in the economy. Note that in this case $\text{rank}(R) = 2 = S$, and the market structure is complete.

**Proposition 3.** *If the market structure is complete an individual can trade in financial markets to achieve the same outcome as under full insurance.*

**Proof.** Suppose there are 2 states of nature, in state one the individual remains unemployed, while in state two the individual finds occupation corresponding to agents level of human capital and therefore captures return on the investment. Suppose there are two assets with return matrix $R$. The individual purchases $z_1$ shares of asset one and $z_2$ shares of asset two and chooses $z_1$ and $z_2$ in such a way that

$$Rz = b,$$

(26)
where $z$ is column vector with components $z_1$ and $z_2$ and column vector $b$ is defined by $b = (d, 0)$, where $d$ is the insurance claim given in Proposition 1. By assumption, market structure is complete, therefore $R$ has full rank, which implies that solution to (26) exists. Furthermore, since in our case $S = K = 2$, matrix $R$ is a square matrix, therefore it possesses and inverse and we can write the solution as

$$ z = R^{-1}b, \quad (27) $$

which concludes the proof.

So far we did not compare the price of the portfolio we constructed in the proof of Proposition 3 with the cost of insurance we found in Proposition 1. Let vector of asset prices be $q = (q_1, q_2)$ Then the cost of the portfolio is

$$ c = qR^{-1}b, \quad (28) $$

where $q$ is interpreted as a row vector. The cost of this portfolio does not exceed the cost of the insurance, if

$$ qR^{-1}b \leq z. \quad (29) $$
One can interpret the last condition in the following way. Under assumptions of rational expectations and common prior beliefs concerning the asset returns there are no speculative motives to trade in the asset market. Therefore, the only reason the assets are traded is to share risk. Since the only risk in this model is the aggregate risk of unemployment, asset market effectively plays the role of unemployment insurance, provided by the stock issuing firms to individuals. If condition (29) does not hold, then a private firm can enter the market and provide a direct unemployment insurance, effectively shutting down financial markets.

Proposition 3 states that financial markets can allow one to diversify risk to human capital investment. In practice, this can be done if one trains to work in a particular industry, but buys the stock of a competitor. Note, however, that such measures are only efficient if the main source of unemployment risk is aggregate. If, on the other hand, the risk is idiosyncratic, it is unlikely to be captured by assets’ returns and direct income insurance can be needed. Chapman (2006) argued for the role of government in managing aggregate risks. While we do not see fundamental reasons, why income insurance can be provided by private markets, one cannot rely just on the financial markets to do the trick.
4 Conclusions

In this paper we study the effects of income uncertainty on human capital investment by developing a two period life-cycle model. Although the risk income clarify why the rate of returns in human capital are high as stated in Jacobs (2007), we argue that income uncertainty has a negative effect on human capital investment. This paper showed that in the present of full income insurance, investment in human capital exceeds investment in the case where income is uninsurable. If the main source of uncertainty is the aggregate risk, this problem can be avoided or at least substantially mitigated by developing well functioning capital markets but these measures will be insufficient, however in the face of idiosyncratic risk. Additional measures, such as direct income insurance or income contingent loans are necessary. Thus we developed two periods of life-cycle model and added in insurance as a tool to reduce the idiosyncratic risk. We then proceed by proving that financial market can be used to diversify risk in human capital investment and hence replicate the effect of income insurance towards human capital investment. We concluded that the reduction of idiosyncratic risk in human capital investment has increased the investment and proved that income uncertainty has a negative effect on human capital investment.
References


