Countervailing Incentives and Wage-Employment Contracts

Ishita Chatterjee

Abstract: This paper looks at a two party set-up, manager and workers, with two types of workers - high ability and low ability - possessing private information on their preference for leisure. The owner hires a manager to make production decisions and also to design wage-employment contracts to hire workers. The product market setting is that of a monopoly. I show that in case of information asymmetry the more sales oriented is the owner of the firm lesser are the sales expected. The private information of the worker produces distortion in the wage income received but the extent of distortion depends on the owner to manager incentive scheme which is exogenously determined prior to the self-selection game of the workers. I show that different levels of sales orientation give rise to various distortions in the wage rate of the workers. At higher levels of sales orientation the wage of the low ability worker is distorted and at low levels of sales orientation the wage of the high ability worker is distorted. At moderate levels, however, the asymmetry in information does not help as the workers either receive the reservation level of wage or are paid as per the full information rate. On the other hand, expected sales, though distorted away from the full information level at both extremes of wage distortions, show a steady decreasing trend with the decrease in sales orientation of the firm.

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1 Introduction

The central aim of this paper is to re-examine the relationship between sales orientation and realized sales and outputs. By sales orientation, I refer to a firm’s objective function that explicitly incorporates sales maximization as one of the firm’s goals. There is a large class of firms that do not always maximize profit, and many of them place a great deal of emphasis on sales (see Baumol (1958)) or output. It ranges from public sector enterprises (Fershtman (1990), Bos (1986)) to modern corporations. In the case of corporates, shareholders want maximum profit but, for a host of reasons, induce the managers to pay attention to sales (See for example Basu (1995), Fershtman and Judd (1987), Ishibashi (2001), and Sen (1993)). Szymanski (1994) showed that managers are more inclined towards profit maximization if the firm has weaker workers union, and if the union is strong then more sales orientation occurs. Chatterjee and Saha (2013) study bilateral delegation in the context of monopoly and show that such delegation results in underproduction compared to standard monopoly model.

The above class of models exploits a simple monotonic relation between the degree of sales orientation and realized sales, which help to alleviate the moral hazard problem of managers, or enhances strategic advantage in duopoly, as in Fershtman and Judd (1987). In this paper I show that with asymmetric information this monotonic relation may get altered. If the manager faces imperfect information vis-à-vis workers’ outside opportunity (or reservation utility) then screening contracts may require distortions in both wages and outputs from their symmetric information level. These distortions, both in terms of magnitudes and directions, will be sensitive to the extent of sales orientation. I show that with greater sales orientation, the firm may prefer to under-produce, and conversely with smaller sales orientation over-produce, distorting sales in opposite directions compared to their symmetric information levels.

There is a large literature on implicit contracts that has studied effects of asymmetric information on labor contracts. Though the primary concern of this literature was to explain the twin issues of unemployment and insurance, many papers have delved into “adverse selection” contracts (Kahn (1985) and Moore (1985), for example).

Kahn (1985) observes that, when the opportunity cost of the workers with regard to alternative employment possibilities is private information to themselves, the owner
should design some profit sharing schemes or incentive programs to tackle the problem of moral hazard through imposition of penalty or partial wage insurance. He finds that for high productivity workers a high severance penalty guarantees lesser number of quits and pay depends on the realization of the actual value of the worker’s outside productivity. Moore (1985) examines stochastic labor contracts under asymmetric information when uncertainty arises both due to shocks to the firm’s revenue function as also to the individual specific reservation wages of the workers. Moore observes that an optimal contract in this case, may have involuntary retention and thereby over-employment but aggregate employment may not be higher than the Pareto efficient one.

One important assumption in these models is that the wage contracts are designed by the “owners” or the firms are owner-managed firms. I depart from this assumption by making the firm run by a manager whose objective function may involve sales orientation, as was the case in Fershtman and Judd (1987). If the manager offers the wage contract, then I show that the degree of sales orientation matters for the distortions in wage and employment necessary to achieve separation among different types of workers. I also show that managerial incentives can lead in a natural way to a “countervailing incentive” problem. Countervailing incentives arise when the privately informed agent’s incentive to misrepresent changes direction, or, becomes non-uniform.

The term “countervailing incentives” was first coined by Lewis and Sappington in their 1989 extension to the Baron and Myerson (1982) paper where agency problem is viewed for a regulated firm which has private information regarding its cost structure. Baron and Myerson (1982) showed that, when there is a possibility of two distinct cost realization for a firm, the firm has incentive to always overstate its marginal cost to get a greater compensation from the regulator since the regulated price is set in excess of the realized marginal cost. But the incentive to misrepresent may not be unidirectional and there can be instances when the firm’s incentive is to understate its cost of production so as to make the regulator absorb a greater part of any operating losses.

This is what the Lewis and Sappington (1989) paper observes of the optimal regulatory policy when the fixed cost of the firm is negatively correlated with its marginal cost implying that high fixed cost means a high opportunity cost of production in terms of alternative opportunities (and profit) foregone for an efficient firm. A firm
may understate production cost (low marginal cost meaning efficient production) to convince the regulator that it has a high opportunity cost and may, thus, obtain greater compensation from the regulator. On the other hand, overstating costs will induce the regulator to cut down output below the efficient level and a low cost firm will thus enjoy a greater profit. The paper shows that there is a critical production cost, below which a firm has incentive to overstate and above which understate its true production cost and, thereby, distorting performance both above and below the efficient levels. Thus, for certain cases, an inefficient firm mimics an efficient one and, for others, an efficient one chooses to represent itself as inefficient in order to reap the information rent on the realization of its private information. Similar incentive to misrepresent has been analyzed in the regulation literature by Maggi and Rodriguez-Clare (1995), Laffont and Tirole (1990); in the literature on international trade (Feenstra and Lewis (1991) and Brainard and Martimort (1996)), on public economics (Jeon and Laffont (1999)) and on corruption (Saha (2001)).

The Lewis and Sappington (1989) paper, however, abstained from including any agency relationship within the firm. There are literature on labor contracts where it is the firm who does not have complete information about the production process. The characteristics of the employees of the firm may be private information to themselves and unknown to the owner of the firm.

I will be introducing the workers in a monopoly firm where the workers’ preference parameter relating to leisure is their private information. Moreover, this parameter also renders their utility from outside labor market unknown. To elaborate, the outside labor market offers a fixed-wage-fixed-work hours contract regardless of the workers disutility parameters. In other words, the market does not price-discriminate. While the inside contract written by the manager may offer Pareto improvement (through price discrimination), the manager’s ability to extract surplus is restricted by the fact that he does not observe the workers’ utility either from current employment or from the outside market.

The net utility of a worker (in excess of reservation utility), however, will increase in the leisure parameter, only if the contract specifies a lower employment than the outside market. This creates an incentive for the workers to overstate their marginal utility of leisure. As is well known for typical adverse selection models, such incentives to overstate are offset by reducing employment. On the other hand, the net utility of the worker may rise with the leisure parameter, if the hours to be
worked exceed the work hours mandated in the outside contract. Here, the screening contract will require over-employment. That which possibility occurs depends on the degree of sales orientation.

I show that associated with lower (higher) degrees of sales orientation is the case of under-employment (over-employment). There is also an intermediate situation, where neither over-employment nor under-employment occurs. Thus, asymmetric information can alter the monotonic relation between sales orientation and sales.

2 The Delegation Model

The owner delegates output decisions to the manager and offers him an incentive \( I \). The incentive structure, as in Fershtman and Judd (1987), henceforth FJ, is a linear combination of profit \( \Pi \) and sales \( S \), weighted by the degree of ‘sales orientation’ of the firm\(^1\). Here, \( (1-\beta) \) is the weight on sales and can be termed as the ‘degree of sales orientation’. This \( \beta \) [thereby, \( (1-\beta) \)], is exogenously determined in my model and fixed between 0 and 1. Formally, \( I = \beta \Pi + (1-\beta)S \) where \( \Pi \) denotes profit and \( S_i \) denotes sales of the representative firm. \( \beta \) is exogenously determined\(^2\) prior to the self selection game of the workers by negotiations between the owner and the manager. I further assume that there is no punishment element in the incentive scheme, that is, the manager is not subject to a reduction in monetary pay if he emphasizes on maximization of certain performance measures that are unfavorable to the owner. Hence \( 0 \leq \beta \leq 1 \).

\(^1\)Here I am implicitly assuming that there is no memory in the incentive scheme. Current incentive is affected by current level of profit and sales. Past financial performance, thereby, has no influence on managerial compensation. The executive compensation literature has, however, looked at the dynamic structure of pay-performance relationship. Joskow and Rose (1994) has shown empirically that both past and present financial performance have significant impact on current compensation of the CEO. However, the effect of past performance is not infinitely persistent but die down considerably in about a couple of years. The ‘no memory’ assumption takes care of the possibility of an adaptive standard and allows me to use a fixed standard of performance.

\(^2\)The weight \( \beta \) may be based on the budget standard, prior year standard, discretionary standard, peer group standard or economic value added (see Murphy (1999) for the categorization). Performance targets maybe fixed “subjectively by the board of directors (our owner) following a review of the company’s business plans, prior year performance, budgeted performance, and a subjective evaluation of the difficulty (faced by the manager) in achieving budgeted performance”, Murphy (1999). The actual managerial remuneration being a function of \( \beta \), fixing the optimal \( \beta \) in the very first stage implies that the expected managerial remuneration is fixed at the managers’ opportunity cost. Hence, the marginal cost of production for the firm is determined by the structure of the managerial incentive defined at the onset and the leisure preference structure of the workers.
2.1 The Production Structure

The owner delegates to the manager the task of writing a screening contract with the workers, specifying wage and employment. The manager, however, does not know the type \( \theta \) of the worker which is private information to the latter. In this situation, the wage contract has to be designed such that there is self selection of the workers in a way that also minimizes the screening cost for the manager. Here lies a possibility for the worker to misrepresent his type, in line with the countervailing incentives literature, hence producing a distortion in the wage and in the expected sales of the firm. In this situation, the manager will form a contract that has a revelation mechanism such that the worker will always respond truthfully and, at the same time, the worker’s minimum utility requirement will be satisfied.

The linear market demand function follows the structure as defined in the previous chapter with \( p = a - x \); the production function having the CRS structure of \( x = l \) where \( l \) is the labor input (the only input) for production. The manager maximizes a FJ type of incentive \( I = \beta \Pi + (1 - \beta)S \). The game is played as follows:

- **Stage 0**: \( \beta \) is exogenously determined.
- **Stage 1**: The manager offers a wage-employment contract to the worker and maximizes his incentive.
- **Stage 2**: Production takes place according to the contract.

2.2 The Worker

A critical assumption regarding the worker is that his leisure preference parameter \( (\theta_i) \) is private information to himself with \( i = 1, 2 \) indicating two possible types of effort level. The worker may be of a high effort \( (\theta_1) \) type with probability \( (p) \) or he may be of a low effort \( (\theta_2) \) type with probability \( (1 - p) \). The type-dependent utility of the worker as a function of the wage income and his effort is:

\[
U_i = w_i + \theta_i(T - l_i), \quad i = 1, 2
\]

where \( \theta_i \) is the coefficient of leisure with a higher \( \theta_i \) signifying higher leisure preference and, hence, lower work effort. Utility is derived from wage income \( w_i \) and leisure with utility increasing in both. \( T \) is the total hours available for work and \( x_i \) denotes the output produced by worker of type \( i \) by working \( l_i \) hours (with \( l_i < T \)).
Thus $T - l_i$ is the amount of leisure available to the worker of type $i$. I assume $\theta_2 > \theta_1$ meaning that worker of type $\theta_2$ is the low effort type and gives more weight to leisure.

A wage-employment combination of $(w_i, l_i)$ is feasible for worker of type $i$, if $U(w_i, l_i) \geq \bar{U}_i$ where $\bar{U}_i$ denotes the reservation level of utility. The reservation utility is given by

$$\bar{U}_i = w + \theta_i(T - \bar{x}), \quad i = 1, 2 \quad (2)$$

This reservation utility level of the worker depends on his type $\theta_i$ and the magnitude of $\bar{x}$ and $\bar{w}$. This $\bar{x}$ is the competitive labor demand; that is to say, if he was to seek employment outside this firm he would have to supply at least this amount of labor and be getting a competitive wage of $\bar{w}$. The net utility to a worker of type $i$, over and above his reservation level, can thus be given as $U_i - \bar{U} = w_i - \bar{w} + \theta_i(\bar{x} - x_i)$.

Let me now look at the manager’s problem in the perfect information case as it will be used as the base for comparing the different results under asymmetric information.

### 2.3 Full Information Case:

The manager’s problem is to maximize his given incentive:

$$\max_x I = \beta \Pi + (1 - \beta) S \quad (3)$$

$$\Rightarrow \max_x (a - x)x - \beta w \quad (4)$$

$$s.t. \quad U = \bar{U}. \quad (5)$$

where the effort level $\theta$ of the worker is known to the manager. There being full information regarding the characteristics of the worker, the manager is able to keep him to the reservation utility level. This gives

$$x^F = \frac{a - \beta \theta}{2}$$

$$w^F = \bar{w} + \theta(x - \bar{x}).$$

Now, the solution of $x$ as a function of $(\beta, \theta)$ may exceed the reservation level of employment or may be lower than that. For different values of $\theta$ i.e. $\theta_1, \theta_2$, the
respective optimum level of employment i.e. $x^F_1, x^F_2$ may lie on the same side of $\bar{x}$ or may lie on either sides $^3$. The three possibilities are graphically illustrated as follows.

Fig 1 presents a case when $\theta_i$ is such that both the possible employment outcomes lie on the higher side of the reservation level of employment. $w^F_1$ and $w^F_2$ represent the indifference curves of the workers, under full information, which denote the net utility gained over and above the reservation level. $\Pi_1$ and $\Pi_2$ are the iso-incentive curves of the manager for $i = 1, 2$ respectively. The full information solution is reached at the usual tangency conditions of constrained maximization at A (for $\theta_1$) and B (for $\theta_2$). It is easy to observe that this solution is not incentive compatible as type 1 will be strictly better off choosing B instead of A i.e. keeping the utility of type 2 fixed, if the worker of type 1 has a rise in utility from C to B. In Fig 2 the two possible solutions lie on either side of he reservation level and in Fig 3 the optimum employment for both types is below the reservation level.

Thus there are possibilities of regime shifting depending upon whether the optimum employment ($x_i$) as a function of $\beta, \theta$ is greater than, equal to or less than the reservation wage ($\bar{x}$). For the purpose of modeling, I have to specify the ranges of $\beta$ for which the manager receives some positive incentives under the different regimes of employment possibilities$^4$.

## 3 Asymmetric Information

In the absence of full information, when the effort level of the worker is private knowledge to himself, a menu of offers for the workers is designed by the manager that comprise incentive compatibility (IC) and individual rationality (IR) constraints. The worker chooses from this menu according to his own type. Otherwise, the manager assigns an offer from the menu when the worker announces his type. Let me discuss this problem by taking up each range of employment possibilities.

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$^3$But it is always the case that $x^F_1 > x^F_2$ since $\theta_2 > \theta_1$ always

$^4$Under full employment, if $\bar{x} > x^F_1 > x^F_2$ then $\bar{x} > a - \frac{\beta \theta_1}{2}$

$$\Rightarrow \beta^F_1 \geq \frac{a - 2\bar{x}}{\theta_1}.$$  

Similarly, for $x^F_1 > x^F_2 > \bar{x}$ the full employment level of $\beta$ for the worker of type II is given as

$$\beta^F_{II} \geq \frac{a - 2\bar{x}}{\theta_2}.$$
Figure 1: Full Information (a)

Figure 2: Full Information (b)

Figure 3: Full Information (c)
3.1 Case I : $x_1^F > x_2^F \geq \bar{x}$

The manager’s problem is to maximize his expected incentive (EI) where

$$ EI = pI(\theta_1) + (1-p)I(\theta_2) $$

and at the same time he has to offer wage contracts to the workers that satisfy the incentive compatibility (IC) and individual rationality (IR) constraints. The worker selects the optimal contract as per his type. The manager’s problem is given as:

$$ \max_{x_1, x_2} EI = pI(\theta_1) + (1-p)I(\theta_2) $$

$$ \Rightarrow \max_{x_1, x_2} p[\beta(S_1 - w_1) + (1-\beta)S_1] + (1-p)[\beta(S_2 - w_2) + (1-\beta)S_2] $$

$$ \Rightarrow \max_{x_1, x_2} p[(a - x_1)x_1 - \beta w_1] + (1-p)[(a - x_2)x_2 - \beta w_2] $$

subject to:

$$ w_1 + \theta_1(x - x_1) \geq w_2 + \theta_1(x - x_2), \quad \ldots IC_1 $$

$$ w_2 + \theta_2(x - x_2) \geq w_1 + \theta_2(x - x_1), \quad \ldots IC_2 $$

$$ w_1 + \theta_1(x - x_1) \geq \bar{w}, \quad \ldots IR_1 $$

$$ w_2 + \theta_2(x - x_2) \geq \bar{w}, \quad \ldots IR_2. $$

The RHS of the IC conditions refer to the case where the worker of one type selects the wage contract meant for the other type; the LHS of the IC conditions refer to the case of truthful revelations. In this case, both $IC_1$ and $IC_2$ can not bind together. Nor can $IR_1$ and $IR_2$. In this case $IC_1$ and $IR_2$ bind (the proof is given in Appendix). Therefore,

**Proposition 1:** The optimal outputs are:

$$ x_1^* = \frac{a - \beta \theta_1}{2} $$

$$ x_2^* = \begin{cases} 
\frac{a - \beta \theta_2}{\bar{x}} & \text{if } x_2 > \bar{x}; \\
\frac{a - \beta \theta_2}{\bar{x}} & \text{if } x_2 = \bar{x} 
\end{cases} $$

where $\theta_2^* = \theta_2 + \frac{p}{1-p}(\theta_2 - \theta_1)$ and $\theta_2^* > \theta_2$.

Compared to the full information case, there is underproduction by the low ability worker.

Thus, with the efficient worker mimicking the inefficient one, a situation of under-
employment results in equilibrium. The corresponding wage rates is given as:

\[
w_1^* = \begin{cases} 
  w - \theta_2(x - x_2) + \theta_1(x_1 - x_2) > w_1^F & \text{if } x_2 > \bar{x}; \\
  w_1^F & \text{if } x_2 = \bar{x} 
\end{cases}
\]

\[
w_2^* = \begin{cases} 
  w_2^F & \text{if } x_2 > \bar{x}; \\
  w & \text{if } x_2 = \bar{x} 
\end{cases}
\] (15)

Thus, the efficient worker extracts some sort of information rent \((AA')\) over and above the full information wage rate when the leisure preference of the workers is unknown to firm.

Here the prevailing \(\beta\) is \(\beta_{II} \geq \frac{a - 2\bar{x}}{\theta_2^*}\). Also, since \(\beta < 1\) and \(\theta_2^* > \theta_2 > \theta_1 > \theta_1^*\), we have

\[
\frac{a - 2\bar{x}}{\theta_1^*} < 1
\]

\(\Rightarrow a - 2\bar{x} < \theta_1^*\) (16)

Keeping \(x_1\) fixed at the full employment level, the type 1 worker can represent himself as a type 2 one and incur a gain in utility. Keeping the type 2 worker on the same indifference curve, the type 1 worker can shift his utility higher up along the \(w_2\) curve till it reaches the point B. The incentive compatibility condition \(IC_1\)
has that \( w_1 - w_2 \geq \theta_1(x_1 - x_2) \), which implies that as \( w_1 \) goes up keeping \( w_2, x_1 \)
constant, \( x_2 \) will have to be reduced by an amount \( \epsilon = \frac{\Delta w_1}{\theta_1} \). The solution is arrived at B’ with \( x_2' < x_2^F \). Thus, tendency for the type 1 worker to misrepresent himself as type 2, forces the equilibrium employment for the type 2 worker to a level lower than the full employment one. This results in a situation of underemployment. The vertical distance AA’ gives the wage concession obtained by the type 1 worker as information rent. When the worker of type 1 is kept at the full information level and he has no incentive to mimic the worker of type 2 as the type 2 equilibrium level is already at the reservation level of \( \pi \). In Fig 4, the equilibrium solution is reached at [A,C].

### 3.2 Case II : \( x_1^F > \bar{x} > x_2^F \)

In this case, neither of the ICs bind but both the IRs bind (See Appendix for Proof). Thus the equilibrium solution is the full information solution at [A,B]. Here when \( \beta_{II} < \beta < \beta_{II}^F \) we have \( x_2' < x_2^F \). Hence, it is optimal for worker 2 to choose \( \pi \) till the point when \( \beta > \beta_{II}^F \). Similarly, for \( \beta_{II}^F < \beta < \beta_{II} \), again it is optimal for worker 1 to choose \( \pi \) till the point when again \( \beta > \beta_{II} \).

### 3.3 Case III : \( \bar{x} \geq x_1^F > x_2^F \)

In this case, only IR$_1$ and IC$_2$ bind (See Appendix for the proof). Here, the type 2 worker has the incentive to mimic the type 1 worker, keeping the type 1 worker at the full information utility level and increasing his utility by the vertical distance BB’. The equilibrium employment for the type 1 worker is thus raised from the full information level and his optimal employment level can be pushed up, as in Fig 5, to the reservation level to reach a solution at [C,B].

**Proposition 2:** The optimal outputs are:

\[
x_1^* = \begin{cases} 
\frac{a - \beta \theta_1}{2} & \text{if } \pi > x_1; \\
\bar{x} & \text{if } \pi = x_1 
\end{cases}
\]

\[
x_2^* = \frac{a - \beta \theta_2}{2}
\]

(18)

where \( \theta_1^* = \theta_1 - \frac{1}{p}(\theta_2 - \theta_1) \) and \( \theta_2 > \theta_1 > \theta_1^* \).
Compared to the full information case, there is overproduction by the high ability worker.

Here, the inefficient worker misrepresents himself as the efficient worker and the firm mitigates to the informational asymmetry by making the efficient worker work more. Thus, the situation now is one of over-employment. The respective wages are given as:

\[
\begin{align*}
    w^*_1 &= \begin{cases} 
    w^F_1 & \text{if } \bar{x} > x_1; \\
    w^* & \text{if } \bar{x} = x_1 
    \end{cases} \\
    w^*_2 &= \begin{cases} 
    \bar{w} + \theta_2(x_2 - x_1) - \theta_1(\bar{x} - x_1) > w^F_2 & \text{if } \bar{x} > x_1; \\
    w^F_2 & \text{if } \bar{x} = x_1 
    \end{cases}
\end{align*}
\]

Thus, the inefficient worker by pretending to be an efficient one extracts information rent (BB’) over and above the full information wage rate when the leisure preference of the workers is unknown to firm.

![Figure 5: The equilibrium solution to the case of overproduction](image.png)

Let me denote the \( \beta \) that will prevail in this case as \( \beta_I \). Now \( \bar{x} \) should be at least \( \frac{a - \beta \sigma_i}{2} \). This makes \( \beta_I \geq \frac{a - 2\bar{x}}{\sigma_i} \).

The various ranges of \( x \) for the different regimes of \( \beta \) are presented in Fig.6. The
equilibrium level of employment, in the face of countervailing incentives, has been summarized in Table 1, as per the degree of sales orientation of the firm.

Figure 6: The Loci of production for different ranges of $\beta$

### 3.4 Net Utility

The net utility of the worker $(i, i = 1, 2)$ above his reservation level of utility can be defined as $\Delta U_i = U_i - \bar{U}_i$ where $U_i = w_i + \theta_i(T - x_i)$ and $\bar{U}_i = \bar{w} - \theta_i(T - \bar{x})$, $T$ being the total work hours available. However, the various ranges of $\beta$ result in some distortions in the wage income away from the normal or full information level. The expected wage costs for the 5 regions can be summarized as follows:

In Region 1:

$$\Delta U_1 = (\theta_2 - \theta_1) \frac{a - \theta_2^* - 2\bar{x}}{2}$$

where $\frac{\partial \Delta U_1}{\partial \beta} = -(\theta_2 - \theta_1) \frac{\theta_2^*}{2} < 0$

In Region 5:
Table 1: Equilibrium Employment for the different ranges of $\beta$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>x</th>
<th>Binding Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 - \beta_{II}$</td>
<td>x1 normal</td>
<td>$x^*_1 = \frac{a-\beta \theta_1}{2}$</td>
</tr>
<tr>
<td></td>
<td>x2 underproduction</td>
<td>$x^*_2 = \frac{a-\beta \theta_2}{2}$</td>
</tr>
<tr>
<td>$\beta_{II} - \beta_{II}^F$</td>
<td>x1 normal</td>
<td>$x^*_1 = \frac{a-\beta \theta_1}{2}$</td>
</tr>
<tr>
<td></td>
<td>x2 reservation level</td>
<td>$x_2 = \bar{x}$</td>
</tr>
<tr>
<td>$\beta_{II}^F - \beta_{I}$</td>
<td>x1 reservation level</td>
<td>$x_1 = \bar{x}$</td>
</tr>
<tr>
<td></td>
<td>x2 normal</td>
<td>$x_2 = \frac{a-\beta \theta_2}{2}$</td>
</tr>
<tr>
<td>$\beta_I - 1$</td>
<td>x1 overproduction</td>
<td>$x^*_1 = \frac{a-\beta \theta_1}{2}$</td>
</tr>
<tr>
<td></td>
<td>x2 normal</td>
<td>$x_2 = \frac{a-\beta \theta_2}{2}$</td>
</tr>
</tbody>
</table>

$\theta_2^* \geq \theta_2 \geq \theta_1 \geq \theta_1^*$

$$\Delta U_2 = (\theta_2 - \theta_1)\frac{2\bar{x} - a + \beta \theta_1^*}{2}$$

where $\frac{\partial \Delta U_2}{\partial \beta} = (\theta_2 - \theta_1)\frac{\theta_1^*}{2} > 0$;

elsewhere $\Delta U_1 = \Delta U_2 = 0$.

Here, since $\theta_2 > \theta_1$,

$$\bar{U}_1 = \bar{w} - \theta_1(T - \bar{x})$$

$$\bar{U}_2 = \bar{w} - \theta_2(T - \bar{x})$$

$\Rightarrow \bar{U}_1 > \bar{U}_2$

Fig. 7 plots the net utility levels for each type of worker for the different regions of $\beta$. The change in utility can be interpreted as some kind of information rent to the worker, who has private knowledge about his abilities, to disclose his type in a countervailing incentive situation.

**Proposition 3:** At a very high level of ‘sales orientation’-where $(1 - \beta)$ is high-, the net utility level for the high efficiency worker is above his reservation utility level but starts declining as $\beta$ goes up. Also, at a very high level of ‘profit orientation’ the net utility of the low efficiency worker starts to rise above his reservation utility.

### 3.5 Sales Orientation and Expected Sales

The monopoly exercise generates the production loci for the two workers against different ranges of $\beta$. This $\beta$ can be termed as the “degree of profit orientation”. It
Figure 7: The Net Utility Loci for different ranges of $\beta$

may be, therefore, expected that the higher the owner sets his $\beta$, more the manager will be motivated to maximize profit. But expected sales go up for higher ranges of $\beta$. However, within a defined range of $\beta$, expected sales($ES$) have a negative relation with $\beta$. The expressions for expected sales in the five regions are given as:

\[
ES_1 = \frac{a^2 - \beta^2(p\theta_1^2 + (1-p)\theta_2^2)}{4}
\]
\[
ES_2 = \frac{p(a^2 - \beta^2\theta_1^2)}{4} + (1-p)(a-\overline{x})\overline{x}
\]
\[
ES_3 = \frac{a^2 - \beta^2(p\theta_1^2 + (1-p)\theta_2^2)}{4}
\]
\[
ES_4 = \frac{(1-p)(a^2 - \beta^2\theta_2^2)}{4} + p(a-\overline{x})\overline{x}
\]
\[
ES_5 = \frac{a^2 - \beta^2(p\theta_1^2 + (1-p)\theta_2^2)}{4}
\]

It can be seen that, in all the five regions $\frac{\partial ES}{\partial \beta} < 0 \ \forall i$. That is, in each region, as $\beta$ goes up, the degree of sales orientation (1-$\beta$) goes down and, therefore, expected sales are seen to fall. But, the relationship between the degree of sales orientation and expected sales is not monotonic. For $\beta < \beta_{II}^*$, the expected sales is lower than the full employment level ($ES_3$) and for $\beta > \beta_{II}^*$, the expected sales is above the full employment level. The magnitude of the deviations depend upon the probabilistic
distribution of worker types (i.e. \( p \)) and the relative strength of effort level (i.e. \( \theta_2 - \theta_1 \)). This is illustrated in Fig 8. Formally,

\[
ES_5 - ES_3 = \frac{\beta^2[p(\theta_1^2 - \theta_1^2) + (1-p)(\theta_2^2 - \theta_2^2)]}{4} \geq 0 \tag{20}
\]

\[
ES_3 - ES_1 = \frac{\beta^2[(1-p)(\theta_2^2 - \theta_2^2)]}{4} \geq 0. \tag{21}
\]

This implies that \( ES_5 \geq ES_3 \geq ES_1 \). Now, for \( ES_5 \geq ES_4 \), it must be that

\[
\frac{a^2 - \beta^2(p\theta_1^2 + (1-p)\theta_2^2) - (1-p)a^2 + (1-p)\beta^2\theta_2^2}{\theta_1^2} \geq p(a - \bar{x})\bar{x}
\]

\[
\Rightarrow \frac{a^2 - 4(a - \bar{x})\bar{x} + \beta^2}{\theta_1^2} \geq \beta^2
\]

\[
\Rightarrow (a - 2\bar{x})^2 \geq \beta^2 \theta_1^2
\]

\[
\Rightarrow \frac{a - \beta \theta_1^2}{2} \geq \bar{x}
\]

which follows from the condition that \( x_1^*(\text{Region } 5) \geq \bar{x}(\text{Region } 4) \).

For \( ES_4 \geq ES_3 \),

\[
p(4(a - \bar{x})\bar{x} - (a^2 - \beta^2\theta_1^2)) \geq 0
\]

\[
\Rightarrow \beta^2 \geq \frac{a^2 - 4(a - \bar{x})\bar{x}}{\theta_1^2}
\]

which is true as \( \bar{x}(\text{Region } 4) \geq x_1(\text{Region } 3) \). Similarly, for \( ES_3 \geq ES_2 \)

\[
a^2 - \beta^2\theta_2^2 - 4(a - \bar{x})\bar{x} \geq 0
\]

\[
\Rightarrow \frac{a - 2\bar{x}}{\theta_2^2} \geq \beta
\]

which holds as \( x_2(\text{Region } 3) \geq \bar{x}(\text{Region } 2) \).

Again for \( ES_2 \geq ES_1 \), we have

\[
\beta^2 \geq \frac{a^2 - 4(a - \bar{x})\bar{x}}{\theta_2^2}
\]

which is true as \( \bar{x} \) in Region 2 is greater than \( x_2 \) in Region 1. Thus, \( ES_5 \geq ES_4 \geq ES_3 \geq ES_2 \geq ES_1 \). This leads to the following proposition:
Proposition 4: Deviation of expected sales from the full information level is not monotonic in $\beta$; that is to say, it exceeds the full information level when the weight on sales is lower and it drops below the full information level for a higher degree of sales orientation. For a moderate weight on sales, there is no deviation in expected sales from the full information level.

3.6 Determining $\beta$: RTM Bargaining

In this paper, I have assumed that $\beta$ is determined exogenously prior to the production process and I suggested a possible situation where the owner and the manager settle upon some $\beta$ through negotiation at the beginning of stage 1. It is obvious that, if the owner determined $\beta$ endogenously by profit maximization, then the optimal solution for $\beta$ would always have been $\beta = 1$. That is, if $\beta$ were to vary with production decisions, then the Fershtman and Judd (1987) monopoly solution would have been arrived at with full emphasis on profit maximization.

Let me now examine some situations where $\beta$ can be optimally fixed at some positive value not equal to unity so that we may move away from the standard profit
maximization solution. For illustrative purpose, I will be considering only the underemployment case of \( x_1^F > x_2^F > \bar{x} \) i.e. Case I in this paper. Let the bargaining power of the owner relative to the manager be \( \mu; \mu \in [0, 1] \). They bargain only over the incentive structure. The manager has the right to take production related decisions. Here I present a right-to-manage bargaining game.

### 3.6.1 Bargaining through Sales

The owner is interested in maximizing expected profit and the manager expected sales and the bargaining strength (i.e. \( \mu \)) of the two is known at the onset of the negotiation. The bargaining problem can be written as:

\[
Max_\beta [B = E\Pi^\mu ES^{1-\mu}]
\]

subject to the constraints obtained in the production game

\[
E\Pi = \frac{a^2 - A\beta^2 + 2A\beta - 2[2\bar{w} - \theta_2(2\bar{x} - a)]}{4} \quad (22)
\]

\[
ES = \frac{a^2 - A\beta^2}{4} \quad (23)
\]

\[
A = [p\theta_1^2 + (1-p)\theta_2^2]. \quad (24)
\]

The optimal \( \beta \) is arrived at from the first order condition of the maximization exercise. I, therefore, set \( \frac{\delta B}{\delta \beta} = 0 \) and obtain the value of \( \beta \) for which the production game will be played in the subsequent stages.

\[
\frac{\delta B}{\delta \beta} = 0
\]

\[
\Rightarrow E\Pi^\mu ES^{-\mu}(1 - \mu)\frac{\delta ES(\beta)}{\delta \beta} + ES^{1-\mu}E\Pi^{\mu-1}\mu\frac{\delta E\Pi(\beta)}{\delta \beta} = 0
\]

\[
\Rightarrow E\Pi^{\mu-1}ES^{-\mu}[E\Pi(1 - \mu)\frac{\delta ES(\beta)}{\delta \beta} + ES\mu\frac{\delta E\Pi(\beta)}{\delta \beta}] = 0
\]

\[
\Rightarrow [E\Pi(1 - \mu)\frac{\delta ES(\beta)}{\delta \beta} + ES\mu\frac{\delta E\Pi(\beta)}{\delta \beta}] = 0
\]

\[
\Rightarrow E\Pi(1 - \mu)[-\frac{A\beta}{2}] + ES\mu[\frac{A(1 - \beta)}{2}] = 0
\]

\[
\Rightarrow (1 - \beta)(a^2 - A\beta^2)\mu + \beta(1 - \mu)(a^2 + 2A\beta - A\beta^2 - 2[2\bar{w} - (2\bar{x} - a)\theta_2]) = 0
\]
This gives the optimal solutions for $\beta$. I assume that the second order condition for maximization holds

$$\Rightarrow \frac{\delta^2 B}{\delta \beta^2} < 0 \quad (25)$$

$$\Rightarrow a^2 - 2[2\bar{w} - (2\bar{x} - a)\theta_2] > \frac{A\beta^2}{\mu} [4\mu\beta - 3\mu - 2\beta]. \quad (26)$$

Table 2: Simulating Optimal $\beta$ in RTM Bargaining over Sales

<table>
<thead>
<tr>
<th>$\beta^\mu$</th>
<th>$\mu$</th>
<th>$p$</th>
<th>$\theta_1$</th>
<th>$a$</th>
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<td>5</td>
</tr>
</tbody>
</table>

3.6.2 Bargaining through Output

Another alternative could be where the manager is interested in maximizing the production level i.e. output and bargains with the owner over the incentive structure to that end. The owner, as before, emphasizes on profit maximization. The simualtion table shows that there are some parametric configurations possible for which $\beta \in (0, 1)$.
bargaining problem, thereby reduces to

$$\text{Max}_\beta [B = E \Pi^\mu E \Omega^{1-\mu}]$$

subject to the constraints obtained in the production game:

$$E \Pi = \frac{a^2 - A \beta^2 + 2A \beta - 2[2 \bar{w} - \theta_2(2 \bar{x} - a)]}{4}$$  \hspace{1cm} (27)

$$EO = \frac{a - A \beta}{2}$$  \hspace{1cm} (28)

$$A = [p \theta_1^2 + (1 - p) \theta_2^2].$$  \hspace{1cm} (29)

As before, the optimal $\beta$ is arrived at from the first order condition of the maximization exercise. I, therefore, set $\frac{\delta B}{\delta \beta} = 0$ and obtain the value of $\beta$ for which the production game will be played in the subsequent stages.

$$\frac{\delta B}{\delta \beta} = 0$$

$$\Rightarrow E \Pi^\mu E \Omega^{-\mu} (1 - \mu) \frac{\delta EO(\beta)}{\delta \beta} + E \Omega^{1-\mu} E \Pi^{\mu-1} \mu \frac{\delta E \Pi(\beta)}{\delta \beta} = 0$$

$$\Rightarrow E \Pi^{\mu-1} E \Omega^{-\mu} [E \Pi (1 - \mu) \frac{\delta EO(\beta)}{\delta \beta} + E \Omega \mu \frac{\delta E \Pi(\beta)}{\delta \beta}] = 0$$

$$\Rightarrow [E \Pi (1 - \mu) \frac{\delta EO(\beta)}{\delta \beta} + E \Omega \mu \frac{\delta E \Pi(\beta)}{\delta \beta}] = 0$$

$$\Rightarrow E \Pi (1 - \mu) [-\frac{A}{2}] + E \Omega \mu [\frac{A(1 - \beta)}{2}] = 0$$

$$\Rightarrow 2\mu(1 - A \beta)(a - A \beta) = (1 - \mu)(a^2 + 2A \beta - A \beta^2 - 2[2 \bar{w} - (2 \bar{x} - a) \theta_2] = 0$$

$$\Rightarrow A(1 - \mu) \beta^2 - 2\beta(1 + a \mu) - (1 - \mu) a^2 + 2(1 - \mu)[2 \bar{w} - (2 \bar{x} - a) \theta_2] + 2a \mu = 0$$

This will give the optimal solutions for $\beta$. I assume that the second order condition for maximization holds

$$\Rightarrow \frac{\delta^2 B}{\delta \beta^2} < 0$$

$$\Rightarrow \beta < 1 + \frac{\mu(a - A)}{(1 + \mu)}.$$
Figure 9: Output-Bargaining Solution of $\beta$

3.7 Conclusion

In this paper I show that with asymmetric information the simple monotonic relation between the degree of sales orientation and realized sales may deviate from the full information level. If the manager faces imperfect information vis-à-vis workers’ outside opportunity (or reservation utility) then screening contracts may require distortions in both wages and outputs from their symmetric information level. These distortions, both in terms of magnitudes and directions, will be sensitive to the extent of sales orientation. When the leisure preference of the workers are private information to themselves, there may arise situations when they will have incentives to misrepresent their true characteristics and seek some sort of information rent. I observe that for an efficient worker the information rent obtained is higher if the firm is highly sales oriented and the inefficient worker obtains higher information rent if the weight on sales is lower. For a moderate range of sales orientation, the firm can prevent the extraction of information rent.

The literature on implicit contracts mostly assume that the wage contracts are designed by the “owner” of the firm. I depart from this assumption by making the firm run by a manager whose objective function may involve sales orientation, as was the case in Fershtman and Judd (1987). If the manager offers the wage contract, then the degree of sales orientation matters for the distortions in wage and employment necessary to achieve separation among different types of workers. I also show that managerial incentives can lead in a natural way to a “countervailing

$^6$The simulation table shows that there are some parametric configurations possible for which $\beta \in (0, 1]$. 

22
Table 3: Simulating Optimal $\beta$ in a RTM Bargaining over Output

<table>
<thead>
<tr>
<th>$\beta^m$</th>
<th>$\mu$</th>
<th>$p$</th>
<th>$\theta_1$</th>
<th>$a$</th>
</tr>
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</table>

incentive” problem. That is to say, the distortion in wage and output do not follow a uniform pattern and depend, to a great extent, on the weight on sales placed by the firm. With greater sales orientation, the firm may prefer to under-produce, and conversely with smaller sales orientation over-produce, distorting sales in opposite directions compared to their symmetric information levels.

References


Brainard, S. L. and Martimort, D. (1996). Strategic trade policy design with asym-


A Appendix

A.1 Proof of Proposition 1

Case I (a): $x_1^F > x_2^F > \bar{x}$
In Case I (a): $x_1^F > x_2^F > \bar{x}$. Now if both IRs bind then

$$w_1 + \theta_1(\bar{x} - x_1) = \bar{w}$$  (1)
$$w_2 + \theta_2(\bar{x} - x_2) = \bar{w}$$  (2)

Let $z_i = w_i + \theta_i(\bar{x} - x_i) - \bar{w}$
So, as $\theta_i$ increases, $z_i$ decreases
⇒ agent 1 chooses $x_2$ and is better off.
⇒ IR$_1$ does not bind
⇒ IR$_2$ binds.
If both ICs bind, then from IC$_1$

$$w_1 + \theta_1(\bar{x} - x_1) = w_2 + \theta_1(\bar{x} - x_2)$$  (3)
$$\Rightarrow w_1 - w_2 = \theta_1(x_1 - x_2)$$  (4)

and from IC$_2$

$$w_2 + \theta_2(\bar{x} - x_2) = w_1 + \theta_2(\bar{x} - x_1)$$  (5)
$$\Rightarrow w_1 - w_2 = \theta_2(x_1 - x_2)$$  (6)

But $\theta_2 > \theta_1$. So both (45) and (47) can not be true. If neither ICs bind then from

$IC_1 \Rightarrow w_1 - w_2 \geq \theta_1(x_1 - x_2)$
$IC_2 \Rightarrow w_1 - w_2 \leq \theta_2(x_1 - x_2)$
$$\Rightarrow \theta_1(x_1 - x_2) \leq w_1 - w_2 \leq \theta_2(x_1 - x_2)$$
This means that keeping \((x_2, w_2)\) fixed, if \(x_1\) is increased then
\[
w_1 - w_2 \leq \theta_2(x_1 + \epsilon - x_2) = \theta_1(x_1 - x_2) + \theta_2\epsilon
\]
\[\Rightarrow IC_1 \text{ binds.}\]
\[
IC_1 \Rightarrow w_1 + \theta_1(\bar{x} - x_1) = w_2 + \theta_1(\bar{x} - x_2)
\]
\[\Rightarrow w_2 = w_1 + \theta_1(x_2 - x_1)
\]
and
\[
IR_2 \Rightarrow w_2 = \bar{w} - \theta_2(\bar{x} - x_2)
\]

**In Case I (b):** \(x_1^F > x_2^F = \bar{x}\)

In Case I (b): \(x_1^F > x_2^F = \bar{x}\). Now if both IRs bind then
\[
w_1 + \theta_1(\bar{x} - x_1) = \bar{w}
\]
\[\text{(7)}\]
\[
w_2 + \theta_2(\bar{x} - x_2) = \bar{w}
\]
\[\text{(8)}\]
\[\Rightarrow w_2 = \bar{w}\]
\[\text{(9)}\]

Let \(z_i = w_i + \theta_i(\bar{x} - x_i) - \bar{w}\)

So, as \(\theta_1\) increases, \(z_1\) decreases and \(z_2 = 0\). Hence, IR_1 will not bind and IR_2 will bind at \(w_2 = \bar{w}\). From the IC - s,
\[
\theta_1(x_1 - x_2) \leq w_1 - w_2 \leq \theta_2(x_1 - x_2)
\]
\[\Rightarrow \theta_1(x_1 - \bar{x}) \leq w_1 - \bar{w} \leq \theta_2(x_1 - \bar{x}).\]

This means that as \(x_1\) is increased \(IC_1\) binds
\[\Rightarrow w_1 = \bar{w} - \theta_1(\bar{x} - x_1)\]

**In Case II :** \(x_1^F > \bar{x}\) > \(x_2^F\)

In Case II : \(x_1^F > \bar{x}\) > \(x_2^F\). Now if both IRs bind then
\[
w_1 + \theta_1(\bar{x} - x_1) = \bar{w}
\]
\[\text{(10)}\]
\[
w_2 + \theta_2(\bar{x} - x_2) = \bar{w}
\]
\[\text{(11)}\]

Let \(z_i = w_i + \theta_i(\bar{x} - x_i) - \bar{w}\)

So, as \(\theta_1\) increases, \(z_1\) decreases and as \(\theta_2\) increases, \(z_2\) also increases.

So here both IRs bind.
If both ICs bind, then from $IC_1$

$$w_1 + \theta_1(\bar{x} - x_1) = w_2 + \theta_1(\bar{x} - x_2) \quad (12)$$

$$\Rightarrow w_1 - w_2 = \theta_1(x_1 - x_2) \quad (13)$$

and

$$w_2 + \theta_2(\bar{x} - x_2) = w_1 + \theta_2(\bar{x} - x_1) \quad (14)$$

$$\Rightarrow w_1 - w_2 = \theta_2(x_1 - x_2) \quad (15)$$

But $\theta_2 > \theta_1$. So both (45) and (47) cannot be true. If neither ICs bind then from

$$IC_1 \Rightarrow w_1 - w_2 \geq \theta_1(x_1 - x_2)$$

$$IC_2 \Rightarrow w_1 - w_2 \leq \theta_2(x_1 - x_2)$$

$$\Rightarrow \theta_1(x_1 - x_2) \leq w_1 - w_2 \leq \theta_2(x_1 - x_2)$$

This means that keeping $(x_1, w_1)$ fixed, if $x_2$ is increased then

$$\theta_1(x_1 - x_2 - \epsilon) = \theta_1(x_1 - x_2) - \theta_1 \epsilon < w_1 - w_2$$

$$\Rightarrow IC_1 \text{ does not bind}$$

and keeping $(x_2, w_2)$ fixed, if $x_1$ is increased then

$$w_1 - w_2 < \theta_2(x_1 - x_2) + \theta_2 \epsilon$$

$$\Rightarrow IC_2 \text{ does not bind}.$$ 

### A.2 Proof of Proposition 2

**In Case III (a):** $\bar{x} = x_1^F > x_2^F$

In Case III (a): $\bar{x} = x_1^F > x_2^F$. Now if both IRs bind then

$$w_1 + \theta_1(\bar{x} - x_1) = \bar{w} \quad (16)$$

$$w_2 + \theta_2(\bar{x} - x_2) = \bar{w} \quad (17)$$

Let $z_i = w_i + \theta_i(\bar{x} - x_i) - \bar{w}$

So, as $\theta_2$ increases, $z_2$ increases and $z_1 = 0$. Hence, $IR_2$ will not bind and $IR_1$ will
bind at $w_1 = \bar{w}$. From the $IC - s$,

$$\theta_1(x_1 - x_2) \leq w_1 - w_2 \leq \theta_2(x_1 - x_2)$$

$$\Rightarrow \theta_1(\bar{x} - x_2) \leq \bar{w} - w_2 \leq \theta_2(\bar{x} - x_2).$$

This means that as $x_2$ is increased $IC_2$ binds

$$\Rightarrow w_2 = \bar{w} - \theta_2(\bar{x} - x_2)$$

**Case III (b):** $\pi = x_1^F > x_2^F$

In Case III (b): $\pi = x_1^F > x_2^F$. Now if both $IR$s bind then

$$w_1 + \theta_1(\bar{x} - x_1) = \bar{w} \quad (18)$$

$$w_2 + \theta_2(\bar{x} - x_2) = \bar{w} \quad (19)$$

Let $z_i = w_i + \theta_i(\bar{x} - x_i) - \bar{w}$

So, as $\theta_i$ increases, $z_i$ also increases

$\Rightarrow$ agent 2 chooses $x_1$ and is better off.

$\Rightarrow IR_2$ does not bind

$\Rightarrow IR_1$ binds.

If both $IC$s bind, then from $IC_1$

$$w_1 + \theta_1(\bar{x} - x_1) = w_2 + \theta_1(\bar{x} - x_2) \quad (20)$$

$$\Rightarrow w_1 - w_2 = \theta_1(x_1 - x_2) \quad (21)$$

and from $IC_2$

$$w_2 + \theta_2(\bar{x} - x_2) = w_1 + \theta_2(\bar{x} - x_1) \quad (22)$$

$$\Rightarrow w_1 - w_2 = \theta_2(x_1 - x_2) \quad (23)$$

But $\theta_2 > \theta_1$. So both (45) and (47) can not be true. If neither $IC$s bind then from

$$IC_1 \Rightarrow w_1 - w_2 \geq \theta_1(x_1 - x_2)$$

$$IC_2 \Rightarrow w_1 - w_2 \leq \theta_2(x_1 - x_2)$$

$$\Rightarrow \theta_1(x_1 - x_2) \leq w_1 - w_2 \leq \theta_2(x_1 - x_2)$$

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This means that keeping \((x_1, w_1)\) fixed, if \(x_2\) is increased then

\[
\theta_1(x_1 - x_2 - \epsilon) = \theta_1(x_1 - x_2) - \theta_1 \epsilon < w_1 - w_2
\]

\[
\Rightarrow IC_2 \text{ binds.}
\]

\[
IC_2 \Rightarrow w_2 + \theta_2(\bar{x} - x_2) = w_1 + \theta_2(\bar{x} - x_1)
\]

\[
\Rightarrow w_2 = w_1 + \theta_2(x_2 - x_1)
\]

and

\[
IR_1 \Rightarrow w_1 = \bar{w} - \theta_1(\bar{x} - x_1)
\]